

Analysis of errors due to deficient mastery of prerequisite skills, facts and concepts: A case of financial mathematics

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ABSTRACT

The main aim of the study was (1) to identify errors committed by learners in financial mathematics, and, (2) to understand why learners continue to make such errors so that mechanisms to avoid them could be devised. It has been hypothesised that errors committed by learners in financial mathematics are not due to a lack of prerequisite skills, facts and concepts. Using Newman's Error Analysis as a theoretical framework, a four-point Likert scale and a content-based structured-interview questionnaire was developed to identify the errors committed. The study was conducted by means of a case study guided by the positivists' paradigm where the research sample comprised of 105 Grade10 Mathematics Literacy learners as respondents. A structured-interview questionnaire was used for collecting data, aimed at addressing the main objective of the study. In order to test the reliability and consistency of the items in the questionnaire, Cronbach's Alpha was calculated for standardised items ($\alpha = 0.705$). Data analysis through content analysis and correlation analysis revealed that learners tend to forget to read the instructions (A) and rounding off incorrectly (C), was weakly significant, as $p < .01$ ($r = +.31$). The hypothesis was tested through Analysis of Variance (ANOVA) revealed that errors committed by learners in financial mathematics are not due to the lack of prerequisite skills, facts and concepts, as the variables showed non-significance.

Keywords: Errors, financial mathematics, prerequisite skills, concepts

INTRODUCTION

A Mathematical Literacy (ML) teacher always administers different assessment tasks throughout the course of the year as per requirements of the National Curriculum Statements (NCS). These tasks are administered in order to determine learners' understanding of the concepts taught inside and outside the classroom. According to the Department of Education (2005: 101)

... assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and thereby assist the learners' development in order to improve the process of learning and teaching.

Surprisingly, learners who seem to follow the trend of the lessons, commit errors when working out the tasks assigned. This stimulated the researcher to critique, understand and undertake research to try and find answers as to what are the underlying factors contributing to common errors committed by Grade 10 ML learners in financial mathematics. Financial mathematics accounts for 35% weighting of the topics in the examination, which indicates that it is more valuable in the ML curriculum (Department of Basic Education [DBE], 2011). It encompasses a number of basic mathematical skills such as: interpreting, communicating answers and calculating, number and calculations with numbers. This is where learners lose marks in their assessment tasks. Financial mathematics could be seen as the Application Topic, which according to the DBE (2011: 13) 'contain[s] the contexts related to scenarios involving daily life, workplace and business environment, and wider social, national and global issues that learners are expected to make sense of content and context'. Topics in financial mathematics include: financial documents, tariff systems, income, expenditure, profit/loss, income-and-expenditure statements, budget, interest, banking, loans and investments.

This study adds, in particular, to the small body of research in error analysis in ML. It focuses on the underlying factors related to the errors due to deficient mastery of prerequisite skills, facts and concepts. Teaching strategies that involve more of drill and practice have been replaced by the reformed approaches that recognise that errors form a valuable source of understanding learners' thinking. Teachers find it difficult to escape from learners' errors so it is worthwhile finding out why learners make them.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Error analysis in mathematical education

By pinpointing learner errors in mathematical literacy, the teacher can provide instruction targeted to the learners' area of need. Learners who have difficulty learning mathematical literacy typically lack important conceptual knowledge for a number of reasons, including an inability to process information at the rate of the instructional pace; lack of adequate opportunities to respond; and the lack of specific feedback from the teacher regarding the misunderstanding cited.

Hodes adapted the following table from Nolting (1998: 1), which illustrates five types of errors for word problems.

*Table 1:
Types of errors for word problems*

1. Read errors	The learner cannot read a key word or symbol correctly.
2. Comprehension errors	The learner reads all the words in the problem accurately but does not understand the overall problem or specific terms within the problem.
3. Transformation errors	The learner understands what the problem requires but is unable to identify the operation or the sequence of operations needed to solve the problem.
4. Procedural errors	These include: <ul style="list-style-type: none"> • Placement errors which is incorrect sequencing of digits or alignment of algorithms. • Incorrect steps which is use of steps that are not associated with any operations. • Missing steps where steps necessary to complete a procedure are missing.
5. Encoding errors	A learnersolves the problembutdoes notwrite the solution in an appropriate form.

(Adapted from Nolting, 1988: 1)

The aforementioned types of errors have been used in the identification of learner errors in the content analysis. Brodie (2005: 179) brought into the debate of learner errors 'Situative perspectives: Situative perspectives argue that what a learner says and does in the classroom make sense from the perspective of his/her current ways of knowing and being, his/her developing identity in relation to mathematics and to his/her previous experiences of learning mathematics, both in and out of school. After engaging with learners in class discussions of a particular topic, Brodie developed a coding scheme to categorise learners' contribution (Brodie cited in Khan & Chishti, 2011: 656).

*Table 2:
Brodie's coding scheme to categorise learners' contribution*

Basic Error	An error not expected at the particular grade level, indicates that the learner is not struggling with the concepts that the task is intended to develop, but rather with the other concepts that are necessary for completing the task and have been taught in previous years
Appropriate Error	An incorrect contribution expected at the particular grade level in relation to the task
Missing Information	Correct but incomplete and occurs when the learner presents some of the information required by the task but not all of it
Partial insight	Learner is grappling with an important idea, which is not quite complete, nor correct, but shows insight into the task
Complete correct	Provides an adequate answer to the task or question
Beyond task	Related to the task or topic of the lesson but goes beyond the immediate task and/ or make some interesting connections between ideas

(Brodie, 2005: 177)

Riccomini (2005: 233) brought into perspective (1) *unsystematic errors*: unintended, non-recurring wrong answers which learners can readily correct by themselves; (2) *systematic errors*: though they are recurring wrong response methodologically constructed and produced across space and time, they are symptomatic of a faulty line of thinking that causes them to be referred to as misconceptions. Elbrink (2008: 2) categorises learners' mathematical errors into three main categories: calculation errors, procedural errors and symbolic errors.

*Table 3:
Summary of the above-stated categories of learner mathematical errors*

Error Category	Description
1. calculation errors	• mistakes in addition, subtraction, multiplication and division
2. procedural errors	• occurs when learner computes or applies an incorrect procedure and symbolic errors
3. symbolic errors	• occurs when learners falsely relate mathematical problems that use similar symbols

Elbrink (2008) elaborated each of the categories as: (1) an error of numbers, which she attributes to carelessness and lack of attention. She further suggested the possible solution to the calculating error is incorporation of an error checklist into a regular classroom routine and procedures. This will allow learners to assess themselves and identify repeated errors and mistakes in their work. (2) Learners are

usually taught in drill and practice and so be automated to carry out specific mathematical tasks rapidly and effectively and can be confused for conceptual understanding. Therefore they cannot recognise the importance of applying a procedure correctly.

Procedural errors suggest that learners do not understand the concepts related to the procedure and are unable to build procedure from conceptual knowledge. She suggested the introduction of the concepts before the procedure, concrete manipulation and real-life application. In her elaborate discussion of procedural errors she brings up the importance of threshold concept which forms part of the theoretical framework of this study. Finally (3), learners try to create meaning in the patterns of mathematical symbols and signs that they see in front of them rather than trying to understand. The identification of errors in the content analysis is based on the aforementioned errors. The errors described in Tables 1, Table 2 and Table 3 have been utilised to categorise the identified errors in this study. The procedural errors which were identified during content analysis could be eradicated from learners by means of teaching that is embedded on the threshold. It is strongly associated with the errors due to incorrect association or rigidity of thinking which was stated as the second research question. The researcher chose to name this particular type of error as 'Radatz' as that also describes it.

In the aforementioned discussions, a number of studies by different researchers have been reviewed; this study focused on the Radatz (1979) classification of errors which brings about the underlying factors that can be associated with learner errors. The types of errors discussed below form a fundamental part of the research propositions of this study.

One should, of course, acknowledge that errors are also a function of other variables in the education process which classifies errors according to information processing. Research questions and hypotheses of the study have been formulated based on the Radatz (1979: 164) classification of errors according to individual difficulties of learners.

Mathematical thinking

Studies have shown that mathematical thinking can be described in terms of two distinct but interrelated components: (1) a non-verbal spatial understanding of quantity, and, (2) a verbal understanding that is related to language and symbolic reasoning (Radatz, 1979; Murray, 2012). According to Setati as cited in Tshabalala (2012: 22) 'in order to develop mathematical thinking, learners have to be able to communicate mathematically'. Teachers should encourage learners to use correct mathematical language and avoid oversimplification through the use of everyday English language. The aforementioned description of mathematical thinking can be closely correlated to the learning process of mathematical literacy.

Goswami (2008: 282) states that 'small amounts of training can lead to rapid improvement in the strategic use of rehearsal, with accompanying improvement in recall'. The recall of the basic formula and the relevant algorithms is an important skill as when learning financial skills the use of formula may be required.

Understanding in the learning process of mathematics

Understanding in the learning process of mathematics can be categorised in two, namely: (1) Instrumental understanding is demonstrated by someone who uses rules without understanding (rules such as to divide by a fraction you turn it upside down and multiply), (2) Relational understanding occurs when one has built up a conceptual structure of mathematics.

'Working memory is especially critical to mathematics learning because mathematics learning places frequent demands on working memory' (Cathercole et al., cited in Soendergaard & Cachaper, 2008:15).

Working memory is therefore the system that actively holds multiple pieces of transitory information in the mind, where they can be manipulated. Students must remember intermediate products of calculations in order to solve problems. Interconnected problems are more common in financial mathematics especially in the income, expenditure and taxation sections. An ability to recall the acquired concepts and skills can be regarded as a good working memory; it has been shown to correlate with successful mathematics learning.

'Relational understanding/thinking occurs when one has built a conceptual structure (schema) of mathematics and therefore both know what to do and why when one solves a mathematical problem' (Soendergaard & Cachaper, 2008: 16). For instance, when dealing with simple and compound interest, the interest may be compounded monthly for three years; that then demands rational thinking of the fact that three years is thirty six months in trying to find the value of n . Rational thinking needs to be developed through teaching and learning in the classroom and thus will play a major role in eliminating or reducing the errors committed by learners.

Theoretical Framework

This study was guided by three theoretical frameworks: Polya's problem-solving techniques, threshold concepts/ troublesome knowledge of Meyer & Land (2006), and Newman's error analysis in deconstructing the concept of error analysis.

Polya's problem-solving techniques

Polya developed four basic principles that need to be considered during problem solving. Based on the principles the four steps that need to be followed during problem solving were developed later. These might be useful to the learners' problem-solving techniques and the mathematical thinking concept by educators can also assist in eliminating the errors. These techniques according to Polya (1945) are: 1) Understanding the problem; 2) Devising a plan; 3) Carrying out the plan; and 4) Looking back.

Threshold concepts and troublesome knowledge

'A core concept is a conceptual "building block" that progresses understanding of the subject; it has to be understood but does not necessarily lead to a qualitative view of the subject matter' (Meyer & Land 2006: 4). As in the aforementioned discussion of Polya's problem-solving techniques, success in problem solving does not solely depend on the acquisition of concepts but also depends on the choice of the relevant problem-solving technique.

Deficient mastery of prerequisite skills, facts and concepts has been hypothesised in the current study as one of the underlying factors that contribute to learners committing errors in financial mathematics. According to Ratadz (1979) deficit in basic prerequisites includes ignorance of algorithms, inadequate mastery of basic facts, incorrect procedures in applying basic mathematical techniques, and insufficient knowledge of necessary concepts and symbols.

Newman's Error Analysis

The current study was guided by Newman's Error Analysis technique in the error analysis of learners' work. Newman's Error Analysis (NEA) provided a framework for considering the reasons that underlay the difficulties students experienced with mathematical word problems and a process that assisted teachers to determine where misunderstandings occurred. NEA also provided directions for where teachers could target effective teaching strategies to overcome learners' errors (White, 2010: 129-148). By pinpointing the errors committed by learners in financial mathematics, teaching can be directed towards the correct procedure of solving the identified problem. The Newman's error analysis and follow-up strategies have

helped learners with their problem-solving skills, and teachers developed a much more consistent approach to the teaching of problem solving (White, 2010).

Specific Objective

To be able to reduce and/or eliminate errors committed by learners, both learners and educators need to be able to (1) identify the errors and (2) understand why learners continue to commit the errors and then be able to avoid the identified errors. The research focused on the mechanisms involved in errors as applied in financial mathematics.

Research Questions

1. What are the types of errors learners encounter in financial mathematics?
2. Why do learners continue to commit errors of similar nature from previous given task(s)?

Research Hypothesis

H₀: Errors committed by learners in financial mathematics are due to deficient mastery of prerequisite skills, facts and concepts.

H₁: Errors committed by learners in financial mathematics are not due to deficient mastery of prerequisite skills, facts and concepts.

RESEARCH METHODOLOGY

The study was guided by the positivist paradigm, which included the use of a quantitative approach for the measurement of data in order to discover and confirm causes and effects of errors committed by learners in financial mathematics. The selection of the case purposively included one East London district school; Grade 10 Mathematical Literacy learners, however, the respondents were selected using a simple random sample technique. The researcher considered the accessibility, travel costs and the time frame when choosing this particular school. The researcher ensured that each member of the sample had an equal chance of being selected and the selection of each member was independent of the selection of the next. Readily available class lists from the research site, learners' names were coded (i.e. each name was assigned a 3-digit code such as 000), were used by the researcher to select randomly 105 respondents from the list.

Sample size (n) and justification

This school had five Grade 10 ML classes with 186 learners. There were 104 girls and 82 boys with ages ranging from 14 to 18 years. The researcher adopted the simplified formula by Yamane cited in Israel (2009: 11) for proportions to determine the sample size (n), where e is the level of precision.

$$n = \frac{N}{1 + N(e)^2}$$

Hence the sample size (n) was nearly 105 where, N=186 was the population size and assuming that confidence level is 95% and the level of precision is 0.5.

Data-collection methods

Data were collected by means of structured-interview questionnaires and documentary studies (examiners' reports and other documents on the subject published by the DBE). The documentary analysis was guided by an inquiry on: (1) why do learners commit errors on given tasks in financial mathematics? and (2) errors were due to deficient mastery of prerequisite skills, facts and concepts.

Data-collection instruments

A structured-interview questionnaire was used which is a content-based questionnaire where respondents are expected to work out financial mathematics problems and one set which includes the possible underlying factors related to the different types of errors learners commit. The second questionnaire with rating scale questions using a Likert scale was used to find the underlying reasons as to why learners commit error from the respondents (Grade 10 ML learners of the participating school in the East London district).

Data analysis

In the first questionnaire which is content based, the researcher was guided by the Newman's error analysis in content analysis and identification of errors committed. Quantitative analysis with descriptive statistics, which describe the distribution, the relationship among variables and variability through the use of frequencies was used to analyse the second questionnaire. Statistical Package of Social Sciences (SPSS) version 21 was used for correlation coefficient analysis to measure the relationship between variables of each of the aforesaid research questions. Analysis of Variance (ANOVA) was used for testing the hypotheses of the study. In the data-analysis stage, the researcher avoided the TYPE I and / or TYPE II errors by presenting the data without misrepresenting its meaning. That shows that validity of the study cannot be achieved through tests only but when the results of different tools (i.e. two sets of questionnaires) are analysed concurrently.

RESULTS

Content analysis

A content-based questionnaire afforded the respondents an opportunity to work out financial mathematics problems. From a number of questions the respondents illustrated a number of approaches, some are illustrated below citing some errors identified.

1. If R12 000 is invested at 9.5% simple interest per year, calculate the value of the investment after 4 years and three months.

Figure 1:
Response of learner 1

$$\begin{aligned} 9.5\% \text{ of } R12000 &= \frac{R12000 \times 9.5\%}{100} \\ &= R1140 \\ \text{The simple interest in 4 years} &= R1140 \times 4 \\ &= R4560 \\ \text{Total interest} &= R12000 + R4560 \\ &= R16560 \end{aligned}$$

Handwritten work by learner 1:

$$\begin{aligned} 9.5\% \text{ of } R12000 &= \frac{R12000 \times 9.5\%}{100} \\ &= R1140 \\ \text{The simple interest in 4 years} &= R1140 \times 4 \\ &= R4560 \\ \text{Total interest} &= R12000 + R4560 \\ &= R16560 \end{aligned}$$

Learner 1 worked out 9.5% of R12 000, then added 4 to the answer instead of multiplying by 4 even though the period of investment was 4 years and 3 months not 4. Learner 1 should have multiplied by 4.25 as the period of investment is 4 years 3 months. Calculated total interest was then added to the invested amount which was R12 000 in this regard. Elbrink (2008) classifies this type of error as a *calculation error* as the learner mistakenly used addition instead of multiplication. Nolting (1998) classifies this type of error as a *procedural error* as the learner employed incorrect steps.

Figure 2:
Response of learner 2

$$\begin{aligned}
 A &= P(1+i)^n \\
 A &= R12000 \left(1 + \frac{9.5}{100}\right)^4 \\
 A &= R17251.93141 \\
 A &= R17251.93X
 \end{aligned}$$

Learner 2 used a compound interest formula instead of the simple interest formula as the question required. The value of n was incorrectly calculated 4 years 3 months was supposed to be calculated as $4\frac{3}{4}$, which is

4.25 when written as a decimal. Therefore the value of n in this regard is $4\frac{3}{4}$ or 4.25, and

not 4 as it is illustrated in Figure 2.

Figure 3:
Response of learner 3

$$\begin{aligned}
 A &= P(1+i)^n \\
 A &= R12000 \left(1 + \frac{9.5}{100} \times 4\right)^3 \\
 A &= R31536.364X
 \end{aligned}$$

Learner 3 used the relevant formula but substituted an incorrect value of n , calculated 9.5% of R12 000, multiplied the answer by 4 and missed the fact that the period of investment was 4 years 3 months. Learner 3 committed the similar error as learner 2.

Learner 2 used the formula $A = P(1 + in)$ but then incorrectly substituted the values, which is according to Elbrink (2008) classified as the *calculation error*. The learner substituted the value 4 for n , and this can also account for *procedural error* as *incorrect steps* were followed and *missing steps* identified. The errors identified in the aforesaid learner responses can be classified as *errors due to deficient mastery of prerequisite skills* as learners were introduced to the use of formula in the previous grades (i.e. Grades 8 and 9).

Incorrect association is justified by the use of incorrect steps, where learners add instead of multiplying. The incorrect steps might be attributed to the threshold concept where learners had been taught a particular method and tended to use it even in irrelevant situations.

2. How long will it take R5 100 invested at 9% simple interest per year to yield an amount of R7 854?

Figure 4:
Response of learner 4

$$\begin{aligned}
 A &= P(1+i) \\
 R7854 &= R5100 \left(1 + \frac{9}{100}\right) \\
 \frac{R7854 - R5100}{R5100} &= \frac{R2754}{R5100} (1.09) \\
 1.54 &\times 1.09 \\
 &= 1.7
 \end{aligned}$$

Learner 4 wrote an incomplete formula, even though the learner substituted the formula correctly, but there was a missing component in the formula. The learner was required to determine how long it would take the invested amount to yield a given value (i.e. find the value of n).

Expected correct solution

$$A = P(1 + in)$$

$$R7\ 854 = R5\ 100(1 + \frac{9.5}{100} \times n)$$

$$\frac{R7854}{R5100} = 1 + 0.09 \times n$$

$$1.54 - 1 = 0.09 \times n$$

$$\frac{0.54}{0.09} = n$$

$$6 = n$$

Therefore it will take 6 years for the invested amount to yield R7 854.

When comparing the method used in the expected solution it is clear that the learner did not write the formula correctly, and as such could not obtain the correct answer. This type of error could be classified as an error due to incorrect association or rigidity of thinking. The fact that a component of the formula was missing would be classified as a missing step error.

3. Calculate the value of R9 700 invested at 9.5% per annum compound interest for a period of 3 years.

Figure 5:
Response of learner 5

$$A = P(1 + i)^n$$
$$A = R9700(1 + \frac{9.5}{100})^3$$
$$A = R12735,44404$$
$$A = \cancel{R12735,44404}$$

In answering the aforesaid question, the respondent illustrated errors attributed to their prerequisite skills, facts and concepts that were gained in the previous grades. The majority of the respondents used the formulae in working out the simple and compound interest problems. The formulae were drawn from the previous knowledge as the use of formula is not encouraged by the teaching and learning

programme of the Curriculum and Assessment Programme Statements (CAPS).

Some would use the formula and arrive at the correct answer but some would use the incorrect formula but then arrive at an incorrect answer. Others would use the correct formula but incorrectly substitute the formula and as a result arrive at an incorrect answer. The use of the correct formula could not guarantee the correct answer as some learners would not round off correctly, as the final answer is supposed to be rounded off to two decimal places. Only 31.4% of the learners admitted to always rounding off the final answer to decimal places, but 10.5% of the learners admitted they had never rounded off their final answer. About 2% of the research sample admitted to forgetting to write down the correct answer as displayed by the calculator and that indicated negligence.

Based on the analysis of questionnaire 2 where frequencies on each variable were identified, it could be established that very few learners (24.8%) from the sample agreed to sometimes forgetting to read the instructions but about 49.5% of them maintained that they sometimes rounded off the answer to 2 decimal places whereas 31.4% claimed to always round off the final answer to 2 decimal places. About 56.2% of the sample asserted that they rounded off but incorrectly, with 5.7% declaring that they always committed such an error. About 20.9% of the respondents claimed to forget to write down the correct answer as displayed in a calculator when it was used.

Correlation analysis

Correlation among the different variable related to the reasons as to why learners commit errors in financial mathematics was tested and the SPSS output is illustrated in Table 4. The Pearson correlation and significance (p-value) of the four variables of the underlying factors related to errors due to deficient mastery of prerequisite skills, facts and concepts are illustrated in the subsequent table.

Table 4:

The correlation analysis of each variable of underlying factors related to errors due to deficient mastery of prerequisite skills, facts and concepts

		A	B	C	D
A	Pearson Correlation	1	.146	.305	.119
	Sig.(2-tailed)		.182	.002	.281
B	Pearson Correlation	.146	1	.137	.035
	Sig.(2-tailed)	.182		.455	.726
C	Pearson Correlation	.305**	.137**	1**	.208**
	Sig.(2-tailed)	.002	.455		.019
D	Pearson Correlation	.119	.035	.208	1
	Sig.(2-tailed)	.281	.726	.019	

NOTE: the variables of research question 4 were labelled A - D for the writer's convenience in constructing the following table which summarises the correlation and the significance of the stated variables.

A - I forget to read instructions

B - I do not round off the answer to 2 decimal places

C - I do round off but incorrectly

D - I do not write the answer as shown on a calculator

The correlation analysis was conducted to examine the relationship between learners forgetting to read the instructions (A) and learners rounding off answers to 2 decimal places (B), and the results were non-significant and illustrated by a weak correlation, as $p > .05$ ($r = +.15$). The relationship between learners forgetting to read the instructions (A) and learners forgetting to write down the answer shown by the calculator (D) was not significant and a weak correlation, as $p = .281$ ($r = +.12$). The relationship between learners rounding off answers to 2 decimal places (B) and learners rounding off but incorrectly (C) illustrated non-significant results and a weak correlation, where $p = .455$ ($r = +.14$).

The correlation analysis was conducted to examine the relationship between learners forgetting to read the instructions (A) and rounding off but incorrectly (C), was significant, as $p < .01$ ($r = +.31$) but illustrated a weak correlation between the aforstated variables.

The correlation analysis between learners rounding off answers to 2 decimal places (B) and learners forgetting to write down answers shown by the calculator (D) revealed non-significance, $p = .726$ ($r = +.04$) but illustrated a moderate correlation between the variables.

Examining the relationship between learners rounding off but incorrectly (C) and not writing answers as shown by the calculator (D) revealed significance, where $p < .05$ ($r = +.21$) which illustrated a weak correlation between the two variables.

The correlation analysis was conducted to examine the relationship between learners forgetting to read the instructions (A) and learners rounding off but incorrectly (C). The analysis was significant, $p = .002$ ($r = .31$).

Analysis of Variances

Significance of four variables of research hypothesis was tested using a one-way ANOVA test where the results were illustrated in tables showing the degree of freedom and the levels of significance (p-values) of each variable.

*Table 5:
I forget to read the instructions*

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	4.113	3	1.371	1.652	.182
Within Groups	83.849	101	.830		
Total	87.962	104			

Learners forgetting to read the instructions proved not to be significant, as $F(3,101) = 1.652$ and $p = .182$ ($r = .31$). This indicates that forgetting to read the instructions cannot be related to the aforestated hypothesis.

*Table 6:
I do not round off the answer to 2 decimal places*

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2.185	3	.728	.878	.455
Within Groups	83.777	101	.829		
Total	85.962	104			

Learners not rounding off the answer to 2 decimal places did not have any significance on errors committed in financial mathematics, ANOVA illustrated, $F(3,101) = .878$ and $p = .455$ ($r = .20$).

*Table 7:
I do round off but incorrectly*

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	8.751	3	2.917	3.470	.019
Within Groups	84.906	101	.841		
Total	93.657	104			

Analysis of the hypothesis through the ANOVA test illustrated a significance, where $F(3,101) = 3.470$ and $p = .019$ ($r = .31$), revealed less than 5% Type II error. That indicated that there is an effect of learners doing rounding off but doing so incorrectly impacting upon the type of errors learner commit in financial mathematics.

*Table 8:
When using a calculator I forget to write down the correct answer*

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2.860	3	.953	1.292	.281
Within Groups	74.531	101	.738		
Total	77.390	104			

The aforesaid variable does not have any effect on the hypothesis and that was revealed by the ANOVA test results which indicate that 'learners forgetting to write down the correct answer when using a calculator' was not significant, $F(3,101) = 1.292$ and $p = .281$ ($r = .21$).

Assumption test

Skewness and the Standard deviations were used to test the Normality and Homogeneity assumption.

*Table 9:
Descriptive statistical analysis of variables of errors due to the deficient mastery of prerequisite skills, facts and concepts*

	N	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Std. Error
I forget to read the instructions	105	.920	.915	.236
I round off the answer into 2 decimal places	105	.909	-.899	.236
I do round-off but incorrectly	105	.949	-.328	.236
When using a calculator I forget to write down the correct answer	105	.863	1.089	.236
Valid N (listwise)	105			

ANOVA results in testing the aforesaid hypothesis, one variable 'forget to read the instructions' illustrated a significance level where $p < .05$. All the other variables illustrated significance levels where $p > .05$ which indicated non-significance to the research hypothesis. Based on the illustrated results I had to drop the alternative hypothesis and accept the null hypothesis. I then concluded that the errors learners commit in financial mathematics are not due to the prerequisite skills, facts and concepts.

*Table 10:
Identified learner errors and the associated underlying factors*

Question No.	Identified errors	
1.	Use of incorrect formula: $A = P(1+in)$ instead of $SI = P \times I \times n$ as the question required calculation of simple interest	Application of irrelevant rules or strategies
2.	Use of incorrect formula: $A = P(1 + in)$ instead of using formula: $A = P(1 + i)^n$ as the question required calculation of compound interest	Incorrect association or rigidity of thinking

Question No.	Identified errors	
3.	Use of the correct formula but the components of the formula incorrectly substituted	
4.	Could not comprehend the meaning and effect of the question to the relevant formula	Language difficulties
5.	Employed a relevant formula but could not substitute the components correctly	Deficient mastery of prerequisite skills, facts and concepts

DISCUSSIONS

From the aforesaid correlation analysis there was a relationship between learners forgetting to read the instructions and rounding off incorrectly. There was a reasonable confidence that the relationship may be stronger also in another research population. Rounding off correctly and not writing the answer as shown by the calculator also illustrated a degree of confidence with a probability of 5% errors when tested in another research population.

From the four pairs of variables two proved to be significant and illustrated weak correlations. Therefore 50% of the variables showed a correlation and significance to the tested research hypothesis.

All the above displayed variables indicated a weak correlation. This could be predisposed by a number of factors such as the sample size, sample distribution, the relevance of the questions and/or the respondents' interpretation of the questions.

The types of errors that are related to the aforesaid underlying factors include the deficits in content and problem-specific knowledge for successful performance in mathematical tasks. This is attested by ignorance of algorithms, inadequate mastery of basic facts, application of incorrect procedures and insufficient conceptual understanding. Learners did not admit to sometimes forgetting to read the instructions. Only 24.8% admitted to that. The majority of the learners (63.8%) never forgot to read the instructions.

Based on the results of the study learners always remembered to round off the final answer to 2 decimal places. Only a few ($n = 11$, 10.5%) never rounded off their final answer. Even though they rounded off their final answer, many ($n = 53$, 50.5%) sometimes rounded off incorrectly.

The majority of the respondents claimed they never forgot to write down the correct answer when using a calculator.

The following errors were identified by content analysis:

- The use of formula: learners were not encouraged to use any prescribed formula for both simple and compound interest. Due to previously acquired knowledge, learners would recall previously taught formulae and employ those to calculate simple and compound interest. Herein the enhanced threshold concept proved to be irreversible as described previously. Learners would use the formula but incorrectly. They would use a formula to find a final amount (A) when asked to find simple interest (SI).
- ignorance of algorithms, incorrect procedures in applying mathematical techniques, and insufficient knowledge of necessary concepts and symbols. Learners also forget to read the instructions and rounding off incorrectly.

- When the correct formula was used, the components of the formula were incorrectly substituted. This could be associated with a number of factors such as a lack of working memory as the learner needed to remember intermediate products of calculations and the sequence of steps to be followed in order to arrive at the appropriate answer.

On the basis of the identified errors through the content analysis and as noted by Newman's Error Analysis technique, there are reasons that underlay the difficulties students experience with mathematical problems. Indeed teachers need to determine where misunderstandings occur (Elbrink, 2008; Ratadz, 1979; Tshabalala, 2012; Riccomini, 2005; Brodie, 2005; Nolting, 1998; Department of Education, 2005; Cathercole et al. cited in Soendergaard & Cachaper, 2008). Once the direction of misunderstandings is identified, there is the need to provide directions for where effective teaching and learning strategies could be devised to overcome learners' errors (White, 2010).

In the previous grade learners were introduced to the use of formula when working out some mathematical problems. Learners rounding off but incorrectly showed significance with a moderate correlation to learners forgetting to read the instructions. In using Polya's approach through; (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and, (4) looking back through the learner; the following were the underlying factors related to the errors that due to deficient mastery of prerequisite skills, facts and concepts:

- Learners rounding off but incorrectly showed significance with a moderate correlation to learners forgetting to read the instruction.
- Learners not writing answers as shown by the calculator illustrated significance with a weak correlation to learners rounding off but incorrectly.
- Learners forgot to read the instructions; this could be related to the fact that they felt rushed in tests, or to language difficulties.
- When required to round off the answer; they did round off but incorrectly. Rounding off is supposed to be acquired and mastered earlier in their school years.
- When using a calculator they incorrectly transcribed the value displayed by the calculator. A relationship with rounding off incorrectly was revealed by the correlation test.

Meyer & Land (2006) noted the need for core concept and conceptual building block that progress understanding of subject. The authors argue that it has to be 'understood, but it does not necessarily lead to a qualitative view of subject matter' (Meyer & Land, 2006: 4). Following the data from the current data, the work of Newman (1983), Polya (1945), and Meyer & Land (2006), it appears though that learners acquired those prerequisite skills, but they showed no mastery of those skills. It is evident because learners sometimes used an incorrect formula, substituted the formula incorrectly and rounded off the final answer incorrectly.

CONCLUSIONS

Error analysis may be incorporated in the teacher training curriculum as it will assist in reducing or eliminating learner errors. It will assist educators to be able to identify learner errors, assist learners in eliminating those errors and encourage learners to review the work before submission. Understanding learners' rationale when going through their work can, also assist teachers to institute remedial lessons. Educators need to incorporate error analysis in their lesson designs, as knowledge of why learners commit errors is valuable to the educators as it will help strategies. Learners should be taught to apply Polya's problem-solving techniques. That will train them in applying the techniques to make sure they understand the question before they attempt to answer it; to plan before answering; to answer then review.

RECOMMENDATIONS

Further research studies could be conducted in error analysis in financial mathematics but the focus should be on higher grades (Grades 11 and 12) as learners continue to commit these kinds of errors even in those grades. The study population could be increased where a number of schools could be involved (five or more schools) to increase the extent to which the research findings could be generalised. Error analysis is a topic that has not yet been researched much in South Africa especially in both Mathematics and Mathematical Literacy. More studies need to be conducted so it can provide recommendations to assist educators in their lesson designs in order to assist learners in avoiding the identified errors. This could lead to the increase in learner performance in Mathematics and Mathematical Literacy.

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